**2021 May Day MCM**

Problem A. Research on vaccine production problems

COVID-19 wreaking havoc around the world, bringing great disaster to the world. Countries are developing new coronavirus vaccine to control the epidemic. It is assumed that vaccine production needs to go through CJ1 station, CJ2 station, CJ3 station and CJ4 station. Each process can handle 100 doses of vaccine at one time. In other words, a processing box can hold 100 doses of vaccine and send them to the equipment at the station for processing. Moreover, the production can only be completed after all the four stations are processed in the order of CJ1-CJ2-CJ3-CJ4. In order to prevent confusion in the packaging of vaccines, the production department of Company G stipulates that different types of vaccine cannot be produced at each station at the same time. No queue-jumping is allowed in vaccine production, which means that the order in which vaccines are produced should always be the same, according to the order in which they are placed at the first station. And only when the former type of vaccine leaves a certain station, the latter type of vaccine can enter this station.

There are 10 different types of vaccines to be produced, including YM1-YM10. Each box (containing 100 doses of vaccine) of each type was produced 50 times in a simulated production at each station [for safety's sake](file:///C:\Users\dashuju\AppData\Local\youdao\dict\Application\8.9.6.0\resultui\html\index.html#/javascript:;).  It was found that the time required to produce each box of vaccine of different types at each station was not stable, which was caused by various reasons such as production equipment, vaccine purification, worker production fluctuation, etc. The detailed data are shown in the attachment 1.

Please build a mathematical model and answer the following questions.

**Question 1:** Please make a statistical analysis of the production time of each box of vaccine at all stations, such as mean value, variance, [maximum](file:///C:\Users\dashuju\AppData\Local\youdao\dict\Application\8.9.6.0\resultui\html\index.html#/javascript:;) [and](file:///C:\Users\dashuju\AppData\Local\youdao\dict\Application\8.9.6.0\resultui\html\index.html#/javascript:;) [minimum](file:///C:\Users\dashuju\AppData\Local\youdao\dict\Application\8.9.6.0\resultui\html\index.html#/javascript:;), probability distribution, etc. In order to facilitate the vaccine production company managers can intuitively understand the capacity of the production of vaccines at each station. It provides reference for vaccine production.

**Question 2:** It is very urgent for the vaccine testing department in country U to test 100 doses of YM1-YM10 vaccine. In order to deliver the vaccine as quickly as possible, companies need to plan the sequence of vaccine production to save time, based on producing the vaccine at an average production rate per station. Please establish a mathematical model to determine the production sequence of the vaccine. The initial time is 00:00, calculate the total production time, and fill in the result in Table 1.

**Table1 Results of question (2)**

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| --- | --- | --- |
| Processing sequence（Fill in the vaccine number） | The moment to enter CJ1 | The moment to leave CJ4 |
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**Question 3:** In reality, however, the speed at each vaccine is produced in each station is random. If the company requested that the total vaccine delivery time be reduced by 5% compared to the time in question 2. How to determine the production sequence of the vaccine to maximize the probability of completing this task? We should also study the dynamic relationship between the proportion of shortened time and the maximum probability.

**Question 4:** Now the vaccine company has received 10 types of vaccine production tasks of different sizes, as shown in the attachment 2. The production time of each station should not exceed 16 hours per day because the production machine needs to be overhauled and maintained. To avoid vaccine mispackaging, the manufacturing tasks for each type of vaccine cannot be separated, that is, the production of the same type of vaccine can be completed before the production of another type of vaccine. Please establish a mathematical model to arrange a production plan. How many days can the task be completed under the premise that the reliability is not less than 90%?

**Question 5:** If the vaccine manufacturer plans to select producing partial quantities of vaccine within 100 days. The production time per station cannot exceed 16 hours per day, and the production tasks for each type of vaccine can be properly separated, which means that each type of vaccine can be completed partially. Please establish a mathematical model to arrange a production plan and try your best to obtain greater economic benefits.